Stability of cargo suspension arrangements

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Introduction

A cargo unit that shall be lifted by a crane for changing the transport mode, e.g. for loading to or unloading from a ship, must be reliably connected to the crane hook. This trivial procedure is usually called slinging and carried out according to a pre-planned suspension arrangement only, if the cargo unit in question is of particular weight, shape or value.

A suspension arrangement must be designed for accommodating the specified weight and the geometrical needs of the cargo unit, i.e. considering the location of the lift points on that unit and the position of its centre of gravity. Furthermore, sensible surfaces must be observed where contact with slings is undesirable. Uncertainties about the weight and the position of the centre of gravity should be taken into account.

Quite often the lift points on the cargo unit are positioned below its centre of gravity. This creates a potentially unstable suspension which needs particular consideration by the planner as well as the operator.

Complex suspension arrangements of cargo units may consist of the primary suspension of lifting beams or spreaders connected to the cargo hook, and the secondary suspension of the cargo unit connected to the beams or spreaders. Such arrangements are more sensitive against small transverse deviations of the cargo centre of gravity than purely primary suspensions. They react with a greater tilting angle due to the sideway slewing of the secondary suspension. And they can really capsize.

Certainly more the 99% of all world wide lifting operations are conducted without the need to intricately examine the lifting stability. Yet, it is the small remaining minority which may create headache if the correct mechanics of the issue are unknown and "the proof of the pudding has to be left to the eating". For this reason this paper presents some key points, case studies and solutions for the safe lifting of delicate cargo units.

1. Assessment of suspension arrangements

1.1 Principles of suspension stability

In popular language the term stability is quite often used for designating strength or sturdiness of a technical arrangement. In this paper stability stands exclusively for the quality of a suspension arrangement to remain in a desired, normally upright condition and not to overturn after lifting has commenced.

Suspension arrangements are absolutely stable, if the centre of gravity of the cargo unit is situated below the level of the lifting points on that unit. Such lifting points may be trunnions, eye plates or other connections for slings or shackles. In the case of a belly sling arrangement, the highest points of contact of the slings to the cargo unit indicate the level of lifting points (Figure 1.1).

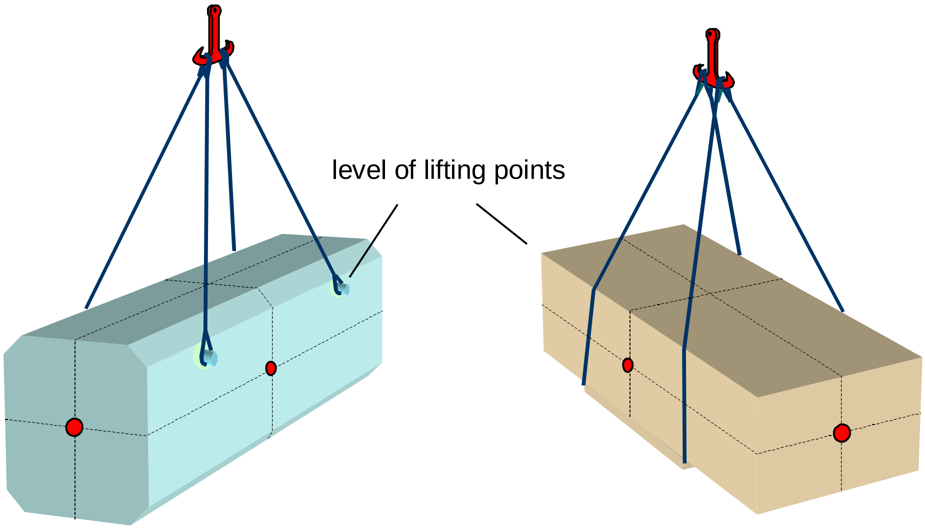


Figure 1.1: Absolutely stable primary suspensions

If the centre of gravity is situated above the level of lifting points (Figure 1.2), the arrangement is potentially unstable. In any of such cases the arrangement should be checked for a positive "metacentric height".

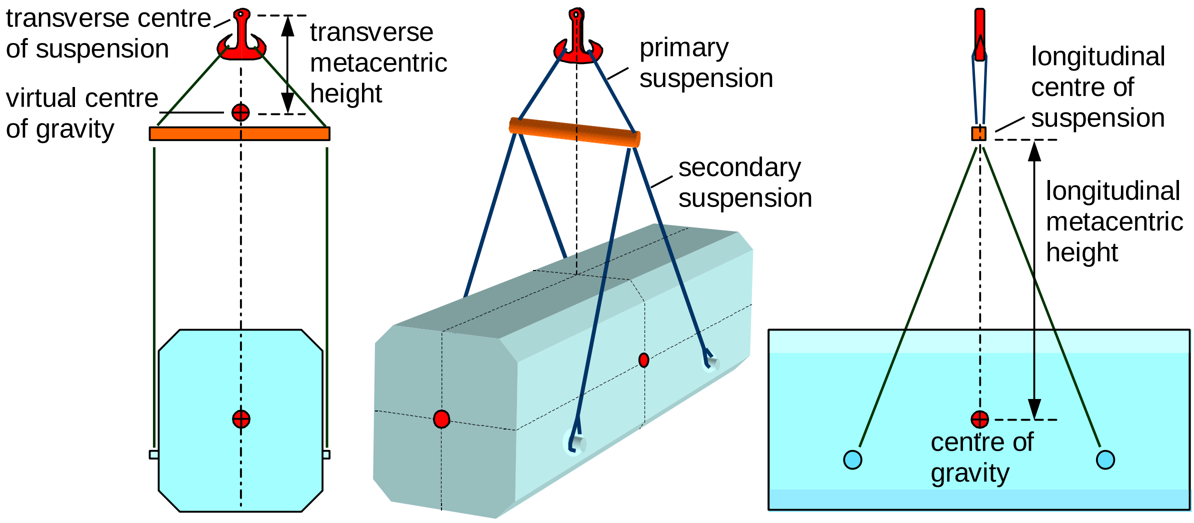


Figure 1.2: Potentially instable primary/secondary suspension arrangement

Figure 1.2 shows a suspension arrangement with different metacentric heights in the transverse and longitudinal orientation. The following definitions are applicable:

Metacentric height	Vertical distance between the centre of suspension and the centre of gravity of the cargo unit or the virtual centre of gravity of the cargo unit, if applicable. The metacentric height is positive if the centre of suspension is above the (virtual) centre of gravity.
Centre of suspension	Pivot point of the suspension for a marginal horizontal offset of the cargo centre of gravity. There may be different pivot points of suspension in the transverse and the longitudinal view of a suspension arrangement. The applicable pivot point is the pivot of the arrangement on the lower end of the vertical line of hoist.
Virtual centre of gravity	Virtual position above the real position of the centre of gravity, caused by the potential lateral slewing of a secondary suspension or by elastic elongation of slings under changing loads.
Primary suspension	Slings connecting the cargo hook directly with the cargo unit or with the beam or spreader.
Secondary suspension	Slings connecting the beam or spreader with the cargo unit.
Range of suspension stability	Angular range of a stable suspension.

Remark for nautical professionals: *There is an immediate analogy between the suspension stability and the stability of a ship and her metacentric height GM. The centre of suspension corresponds to the metacentre of the ship. The virtual centre of gravity in a suspension arrangement corresponds to the virtual centre of gravity of a ship obtained by the KG-correction for liquid free surfaces in tanks, i.e. the slewing secondary suspension has an effect similar to that of the travelling liquid in a partly filled tank.*

The determination of the virtual position of the centre of gravity requires a geometrical analysis of a combined primary/secondary suspension using the following parameters:

- primary suspension angle ϕ,
- height of primary suspension v,
- secondary suspension angle γ,
- height of secondary suspension s,
- distance of centre of gravity from level of lift points z.

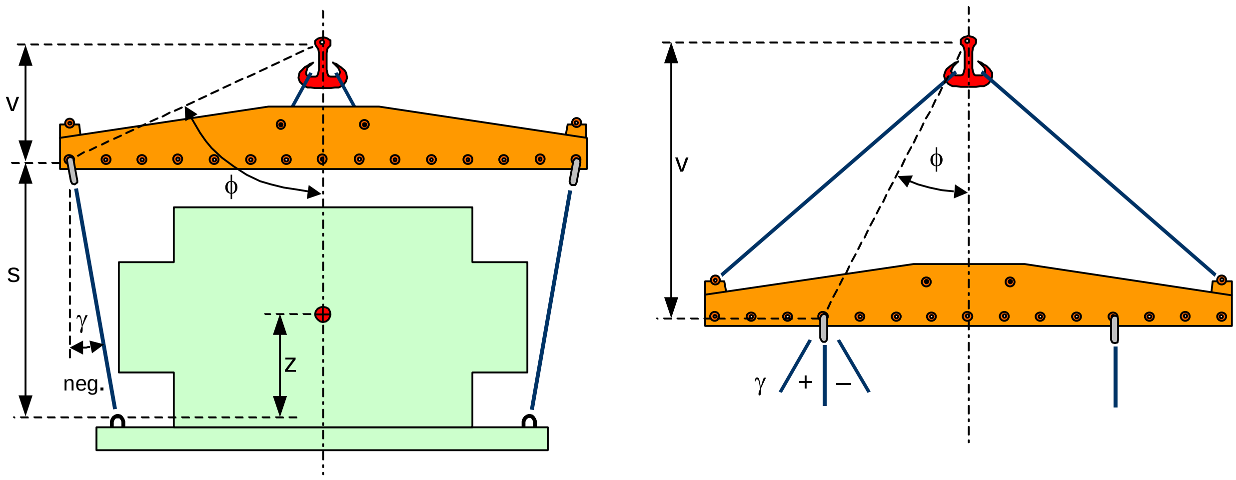


Figure 1.3: Definition of ϕ, γ, v, s and z

Figure 1.3 shows the definition of the geometric parameters. The angle ϕ is measured between the vertical and the line from the centre of suspension to the pivot of the secondary suspension. The angle γ is given the negative sign when the secondary slings point inward.

1.2 Case studies

The phenomenon of suspension arrangements with poor stability is not new and accidents have happened. Former ship borne suspension arrangements suffered from the constraint to keep "slinging heights" as short as possible, because the available "hoisting distances" of the heavy lift derricks were much smaller than those of the today heavy lift cranes. The requirement of keeping the slinging height small caused primary suspension heights of spreaders, which were close to zero by design. This is demonstrated in the next two cases.

1.2.1 Lifting the Santa Maria replica

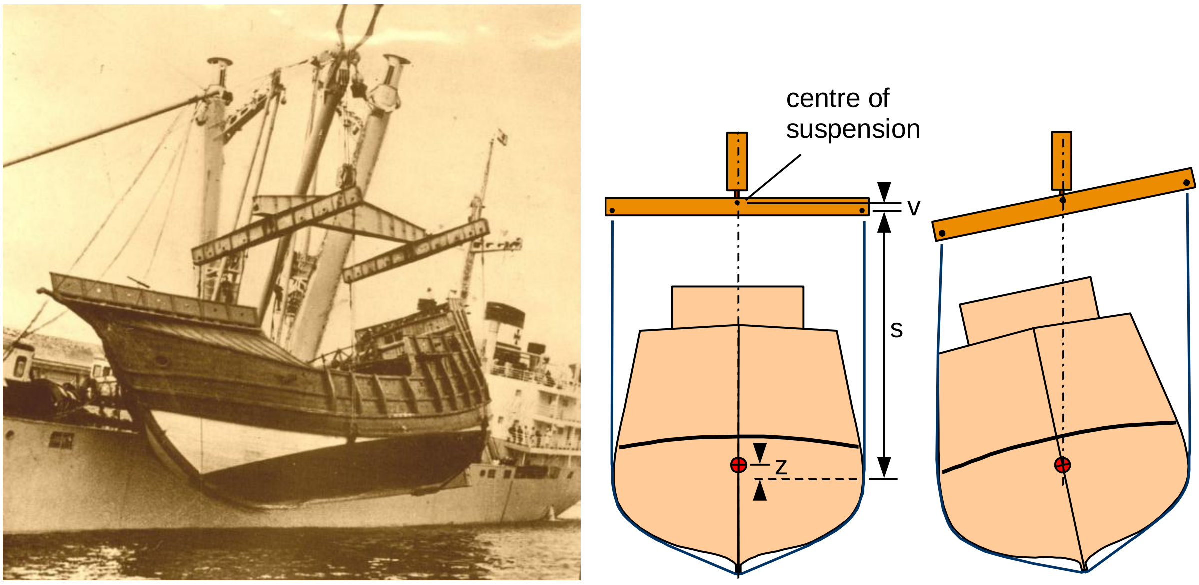


Figure 1.4: Lifting the Santa Maria replica with possibly unstable suspension

The lifting arrangement in Figure 1.4 shows the typical short height v of the primary suspension of the transverse beams at that time. The precise vertical position of the centre of gravity of the replica hull was certainly not known. The metacentric height h of this arrangement may be calculated by:

$$h = v \cdot \left(1 + \frac{m_T}{m_C}\right) - z \;\; [m]$$

h = metacentric height [m]
v = height of the primary suspension [m]
m_T = mass of both transverse beams [t] (about 8 t)
m_C = mass of cargo unit [t] (about 100 t)
z = height of centre of gravity above level of lift points [m]

It appears that the metacentric height of the arrangement is very small or even negative. It therefore remains uncertain whether the tilt of the replica hull results from a very small transverse offset of the centre of gravity or from a negative value of the metacentric height h. In any case the pressure from the slings on the low side of the hull stabilised the arrangement.

1.2.2 Lifting a nuclear reactor

The lifting arrangement for a 360 t unit in Figure 1.5 shows the same short distance v as in the previous case. The arrangement hangs straight, either because the metacentric height is still positive or by the stabilising effect from the contact of the secondary slings to the body of the cargo unit. The metacentric height h may be obtained by the same formula as under case 1.2.1 above.

$$h = v \cdot \left(1 + \frac{m_T}{m_C}\right) - z \;\; [m]$$

Figure 1.5: Lifting a nuclear reactor, close-up of centre of gravity and centre of suspension

1.2.3 Lifting a catamaran

The lifting arrangement in Figure 1.6 has a greater distance v due to the sling connection from the hook to the lifting beam. But there is also a great distance z creating the risk of a very small or even negative metacentric height of the suspension.

The unit did not hang straight and was only stabilised by sling contact. There was no damage to the hull due to the use of soft slings in the secondary suspension. The metacentric height h of the suspension arrangement may be obtained by the formula:

$$h = v \cdot \left(1 + \frac{m_T}{m_C}\right) - z \ \ [m]$$

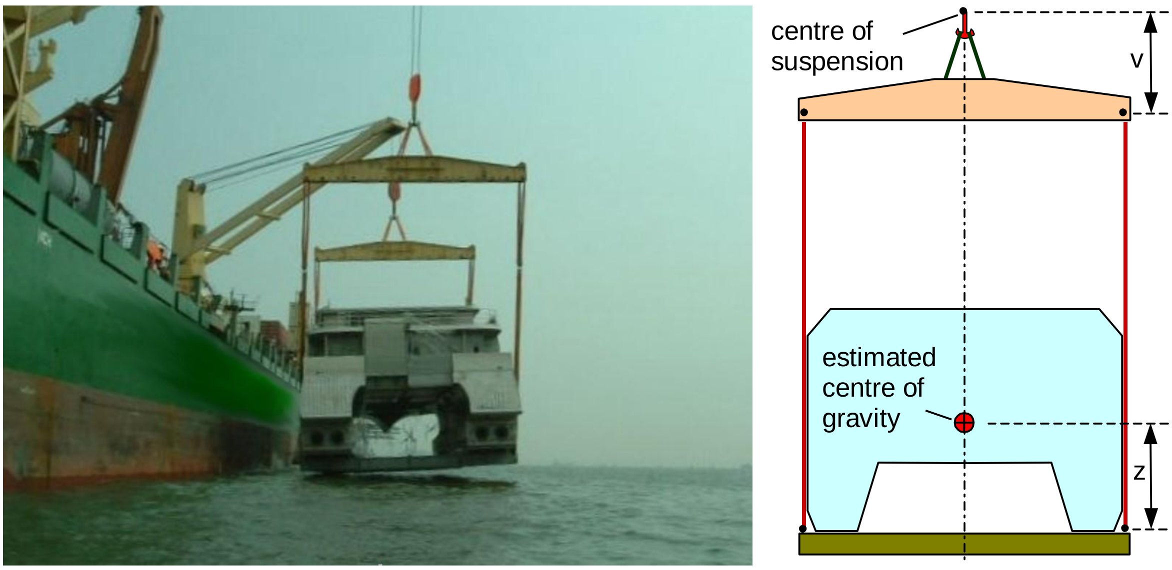


Figure 1.6: Lifting a catamaran hull, estimated centre of gravity

1.2.4 *Arrangement with inclined secondary slings*

The arrangement in Figure 1.7 shows inward inclined secondary slings. Although the height of the primary suspension v is quite large compared to the elevation of the centre of gravity z, the metacentric height of the arrangement is only about 1 metre due to the inward inclined secondary slings. For inclined secondary slings the metacentric height h of the arrangement must be obtained by an extended set of formulas:

$$c = \cos^2\gamma - \left(1 + \frac{m_T}{m_C}\right) \cdot \frac{\sin\gamma \cdot \cos\gamma}{\tan\varphi} \quad \text{(Note: c = 1 for } \gamma = 0\text{)}$$

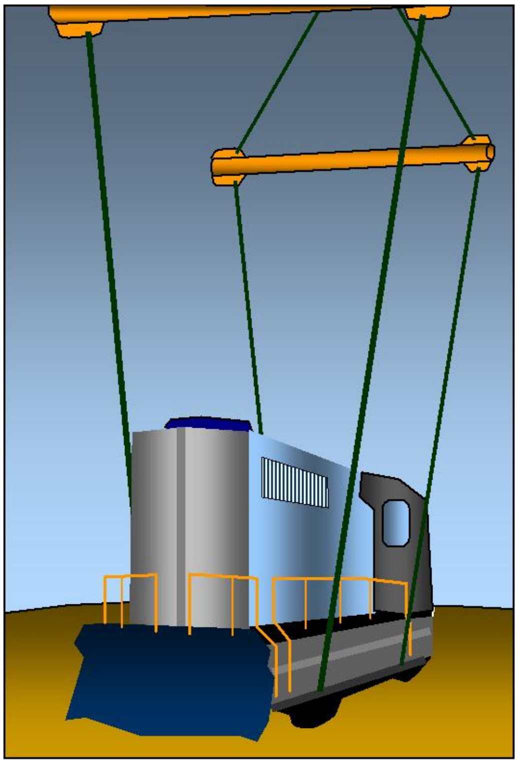
$$h = s \cdot (1 - c) + v \cdot \left(1 + \frac{m_T}{m_C}\right) - z \cdot \left(1 - \frac{c \cdot s \cdot \tan\gamma}{v \cdot \tan\varphi + s \cdot \tan\gamma}\right) \text{ [m]}$$

h = metacentric height [m]
c = conversion factor = $d\gamma / d\phi$
ϕ = primary suspension angle [°]
γ = secondary suspension angle [°]
v = height of the primary suspension [m]
m_T = mass of both transverse spreaders [t]
m_C = mass of cargo unit [t]
z = height of centre of gravity above level of lift points [m]

The estimated parameters in this case are:

v = 3.6 m, s = 7.5 m, z = 1.1 m, ϕ = 26°, γ = - 4°, m_T = 2 t, m_C = 60 t

Results: c = 1.14256, h = 1.0 m



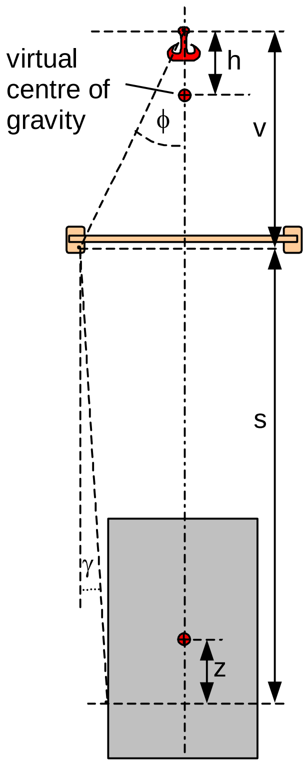


Figure 1.7: Arrangement with inward inclined secondary slings, estimated parameters

This arrangement is quite sensitive against negative angles γ due to the large distance s. An angle γ = - 6° would already create a negative metacentric height.

1.2.5 Dual crane lift with primary suspension

Figure 1.8 shows a dual crane lift with purely primary suspension. Such suspensions are stable as long as the centre of gravity of the cargo unit is below the centre of suspension. The range of stability depends on the width of the slinging base of the unit. The metacentric height of the suspension would normally be calculated by h = (v – z) metres in the longitudinal view, while an unlimited metacentric height is assumed in the transverse view.

However, the stability of such arrangements may be impaired by the flexibility of the slings, because a small offset of the centre of gravity, e.g. to the left, will tilt the arrangement to the left with load increase in the left slings and decrease in the right slings. The resulting changes in length will cause an additional tilt to the left, which may be arithmetically expressed as being induced by an elevated centre of gravity.

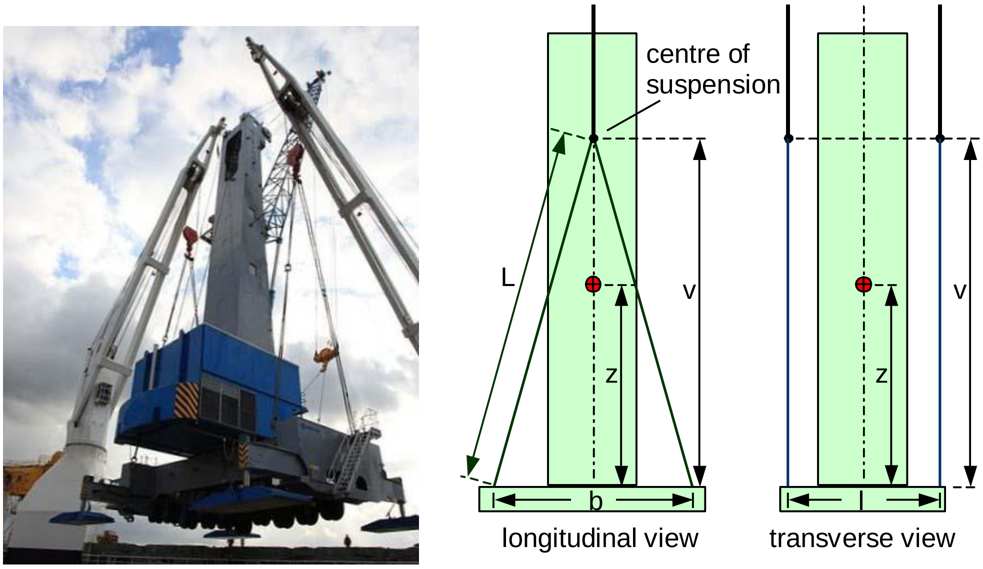


Figure 1.8: Dual crane lift with primary suspension, essential parameters

The relative elongation at reaching their lifting WLL may be attributed to soft slings with ε = 0.023 (= 2.3%) and to wire rope slings with ε = 0.004 (= 0.4%). The formulas for obtaining the metacentric heights are given in chapter 1.3 below. Estimated parameters in this case are:

L = 15 m, v = 14.5 m, b = 7.7 m, l = 5.9 m, z = 8.4 m, ε = 0.023 (polyester), ε = 0.004 (steel)

	zero elongation	steel wire slings	polyester slings
Longitudinal view	h = 6.1 m	h = 5.6 m	h = 3.2 m
Transverse view	h = infinite	h = 141.6 m	h = 17.7 m

The results show satisfactory figures in all cases, but this may change quickly with other parameters, in particular with smaller values of b and l.

1.2.6 Dual crane lifts with primary and secondary suspension

Figure 1.9 shows three versions of suspension arrangements with identical overall slinging height but different shares of primary and secondary suspension. The level at M indicates the centre of suspension, while G* is the virtual centre of gravity. The metacentric height decreases with the share of the height of the primary suspension. The arrangement on the right side is definitely unstable. This leads to the design rule:

In a given slinging height the share of the primary suspension should be as large as possible.

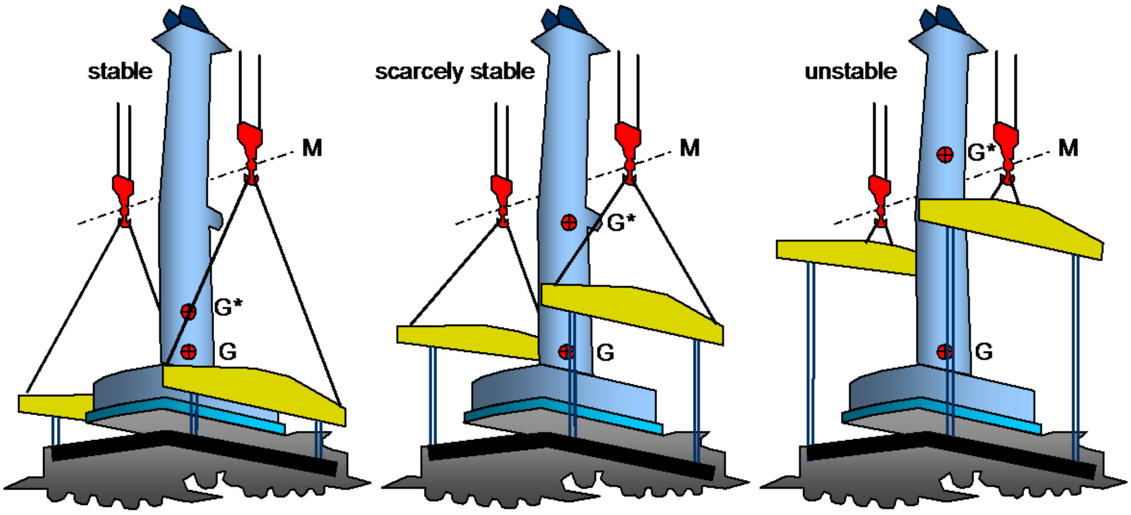


Figure 1.9: Different shares of primary and secondary suspension

1.2.7 Dual crane lift with dramatic capsize

Figure 1.10 shows the result of insufficient suspension stability in the unloading operation of a fully rigged mobile crane in 2010. The metacentric height was obviously marginal and the capsize took place shortly before landing the unit on the jetty. The cause could have been a slight upcoming wind catching the large outrigger of the mobile crane. Gladly no persons were injured. A new crane had to be ordered.

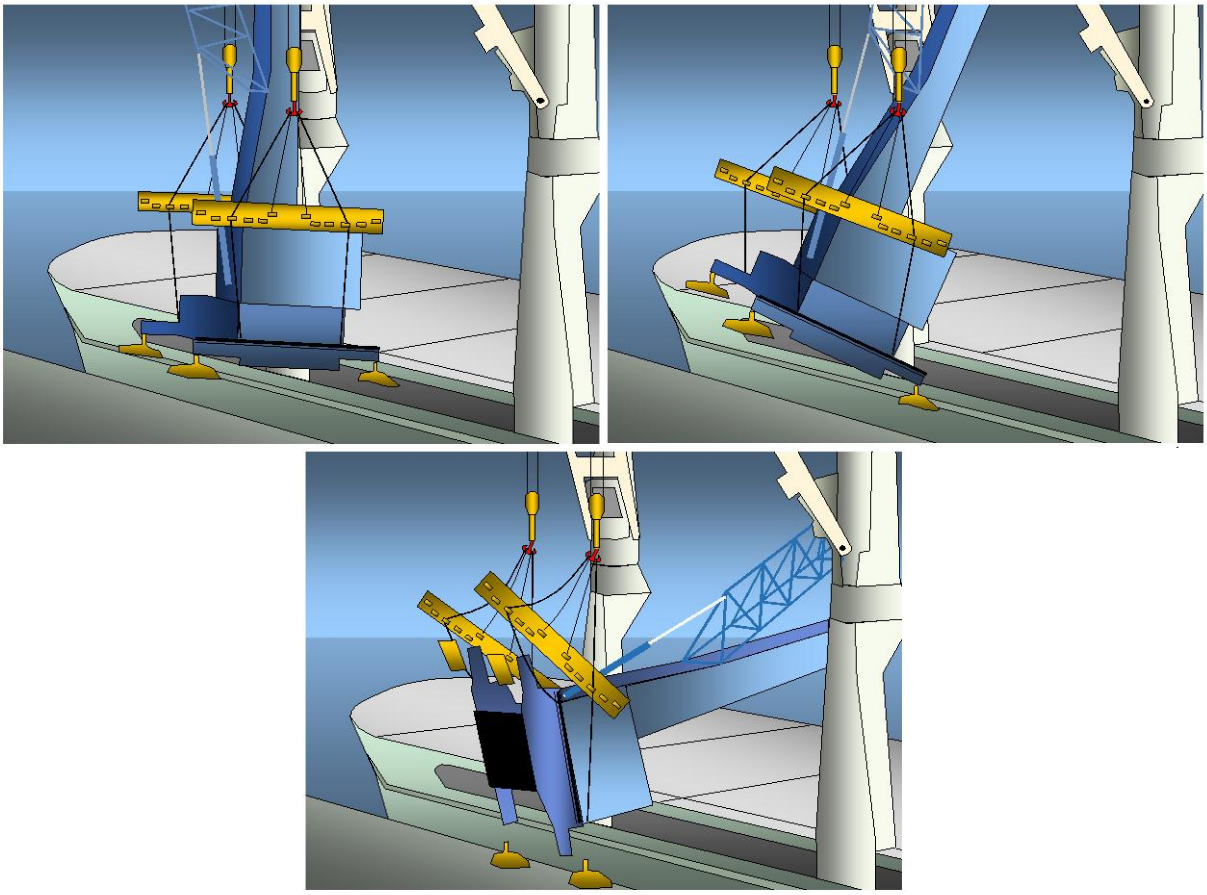


Figure 1.10: Capsize due to insufficient metacentric height of suspension arrangement

1.2.8 Single crane lift with asymmetric suspension arrangement

Figure 1.11 shows an asymmetric suspension arrangement for lifting a heavy dumper truck. The forward suspension consists of a single vertical wire rope grommet that is fastened to a lift point below the centre of gravity of the truck. This forward suspension is definitely unstable with a negative metacentric height h_1 = - z, as shown in the schematic drawing.

The rear suspension consist of a primary and a secondary suspension. The level of lift points is apparently above the centre of gravity, producing a negative value of z with regard to the convention of signs used in this paper. The secondary suspension angle γ has also a negative value. The metacentric height of the rear suspension is obtained by the formulas:

$$c = \cos^2\gamma - \left(1 + \frac{m_T}{m_C}\right) \cdot \frac{\sin\gamma \cdot \cos\gamma}{\tan\varphi}$$

$$h_2 = s \cdot (1-c) + v \cdot \left(1 + \frac{m_T}{m_2}\right) - z \cdot \left(1 - \frac{c \cdot s \cdot \tan\gamma}{v \cdot \tan\varphi + s \cdot \tan\gamma}\right) \text{ [m]}$$

The common metacentric height is obtained by:

$$h = \frac{m_1 \cdot h_1 + m_2 \cdot h_2}{m} \text{ [m]}$$

h = common metacentric height [m]
h_1 = front metacentric height [m]
h_2 = rear metacentric height [m]
c = conversion factor = $d\gamma$ / $d\phi$
ϕ = primary suspension angle [°]
γ = secondary suspension angle [°]
v = height of the primary suspension [m]
m_T = mass of transverse spreader [t]
m_1 = partial mass of cargo unit at front end [t]
m_2 = partial mass of cargo unit at rear end [t]
z = height of centre of gravity above level of rear lift points [m]

The estimated parameters in this case are:

m_1 = 15 t, h_1 = -0.8 m, m_2 = 15 t, m_T = 1 t, v = 2.0 m, s = 3.5 m, z = -1.0 m, ϕ = 45°, γ = - 8°

Results: c = 1.12764, h_2 = 3.05 m

$$h = \frac{15 \cdot -0.8 + 15 \cdot 3.05}{30} = 1.1 \text{ m}$$

The overall suspension is sufficiently stable.

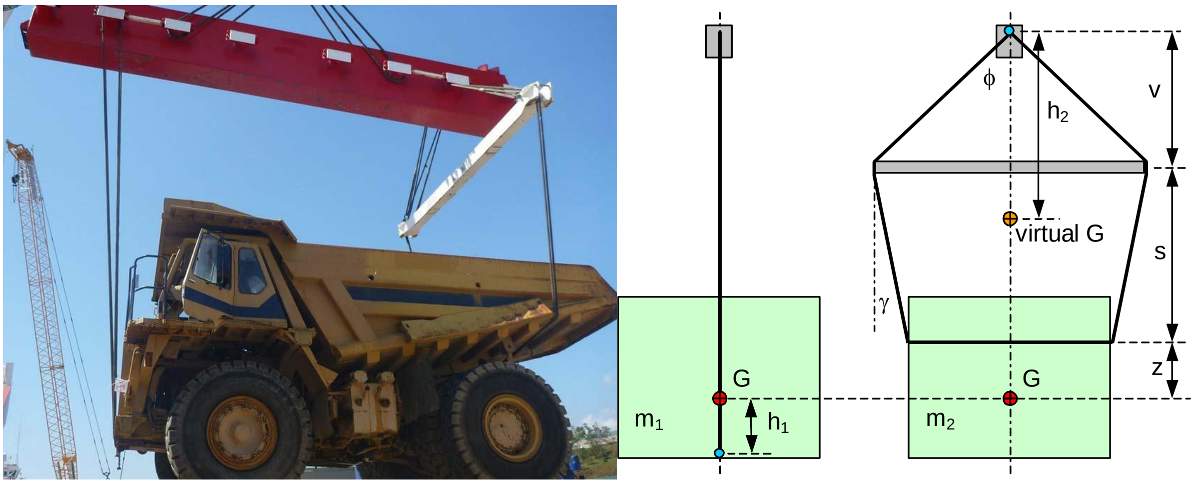


Figure 1.11: Asymmetric suspension arrangement

1.2.9 Dual crane lift with asymmetric suspension arrangement

Figure 1.12 shows a dual crane lift with an arrangement where the centres of suspension and also the virtual centres of gravity of the partial masses are situated in different levels. The common centre of suspension as well as the common virtual centre of gravity may be obtained graphically. This is shown in Figure 1.13.

Figure 1.12: Dual crane lift with asymmetric suspension arrangement

The metacentric heights of the forward and the rear suspension and the common metacentric height are obtained by the same formulas as used in the previous example. The necessary parameters have been estimated with fairly good accuracy as follows:

parameter	left suspension	right suspension
v	6.05 m	6.28 m
s	18.19 m	7.20 m
z	3.54 m	3,54 m
ϕ	27.1 °	22.5 °
γ	-2.1 °	-1.4 °
m_T	1.6 t	1.6 t
$m_{C1,2}$	185.6 t	160.4 t
$c_{1,2}$	1.07083	1.05896
$h_{1,2}$	0.233 m	2.106 m

$$h = \frac{185.6 \cdot 0.233 + 160.4 \cdot 2.016}{346.0} = 1.10 \text{ m}$$

The unit hangs stable, yet the stability margin is small. It could have been improved by elongating the primary suspension on the right side. A capsize was impossible due to the stabilising effect of the long secondary slings contacting the cargo unit.

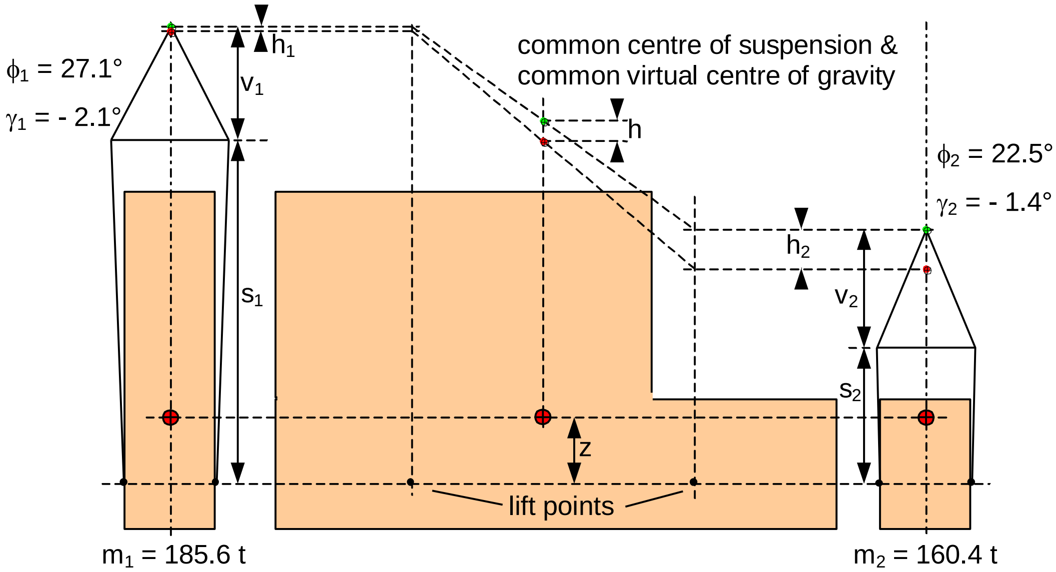


Figure 1.13: Graphical determination of common metacentric height

1.3 Assessment tools

The stability of a suspension arrangement should be assessed, whenever the lift points on a cargo unit are below its centre of gravity. The relevant formulas for assessing a planned arrangement are given below for quick reference.

1.3.1 Primary slings only

Figure 1.14 shows the relevant parameters for assessing a suspension with primary slings only. It should be noted that for reasons of clarity a symmetrical arrangement is assumed.

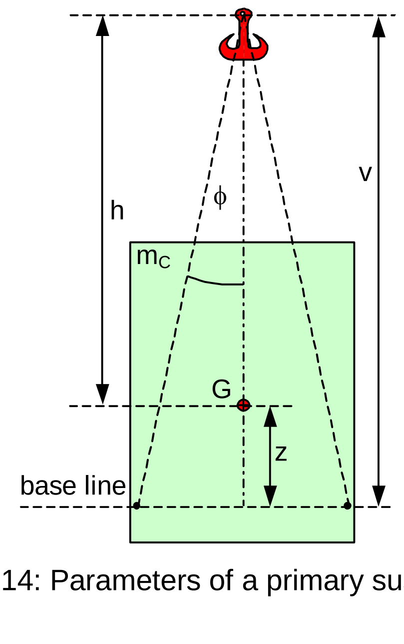


Figure 1.14: Parameters of a primary suspension

Metacentric height

$h = v – z$ [m] Note: z is negative if G is below the base line.

If the angle ϕ < 5° AND $z > v \cdot \tan\phi$, the arrangement is close to a situation where a small disturbance by wind or swell may overwhelm the stability. It is therefore prudent to treat this suspension as a single sling attachment with the pivot at the lift point, unless special precautions are taken to stabilise the hanging unit. Otherwise, the metacentric height is:

$h = - z$ [m]

Use of soft slings

Soft slings have an elongation that is about 6 times greater than that of wire rope slings, both at reaching their respective WLL. This is characterised by the figures of relative elongation for soft slings of $\varepsilon = 0.023$ (= 2.3%) and for wire rope slings of $\varepsilon = 0.004$ (= 0.4%). These figures should be taken as indicative values only. They may vary in practice.

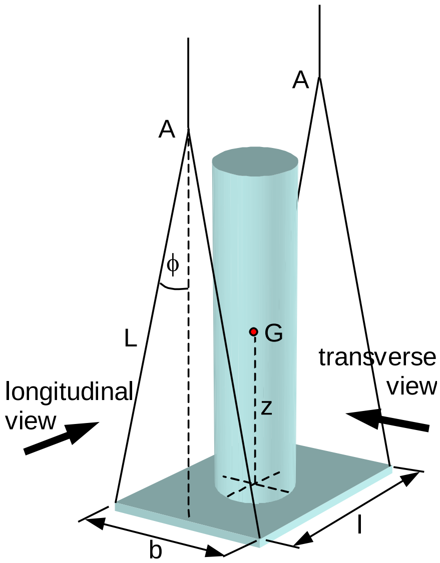


Figure 1.15: Assessment of metacentric height when using soft slings

The metacentric height of such arrangements should be assessed according to the explanation in chapter 1.2.5 by extended formulas:

$$h = v - z \cdot \left(1 + \frac{4 \cdot L^2 \cdot \varepsilon}{b^2}\right) \text{ [m]} \qquad \text{for the longitudinal view}$$

$$h = \frac{l^2}{4 \cdot v \cdot \varepsilon} - z \text{ [m]} \qquad \text{for the transverse view}$$

h = metacentric height [m]
L = length of primary slings [m]
ε = relative elongation of slings at the working load limit WLL
b = distance of lift points in longitudinal view [m]
l = distance of lift point in transverse view [m]
v = height of the primary suspension [m]
z = height of centre of gravity above level of lift points [m]

Range of stability

The range of stability of a primary suspension may be demonstrated by the righting moment of the sling on one side as a function of the tilting angle δ. Such a tilting angle may appear as the result of a transverse offset e of the centre of gravity of the cargo unit.

$$\tan\delta = \frac{e}{h}$$

$$M = \frac{W}{4} \cdot h \cdot \frac{\sin^2\varphi \cdot \cos^2\delta - \cos^2\varphi \cdot \sin^2\delta}{\sin\varphi \cdot \cos\varphi \cdot \cos\delta} \qquad \text{(Note: } M = 0 \text{ at } \delta = \phi\text{)}$$

The initial righting moment in the upright hanging condition with $\delta = 0$ is:

$$M_0 = \frac{W}{4} \cdot h \cdot \tan\varphi$$

δ = tilting angle [°]
M = up-righting moment [kN·m]
M_0 = initial up-righting moment [kN·m]
e = possible transverse offset of the centre of gravity of the cargo unit [m]
h = metacentric height of the suspension arrangement [m]
W = weight of the cargo unit (m · g) [kN]
ϕ = primary suspension angle [°]

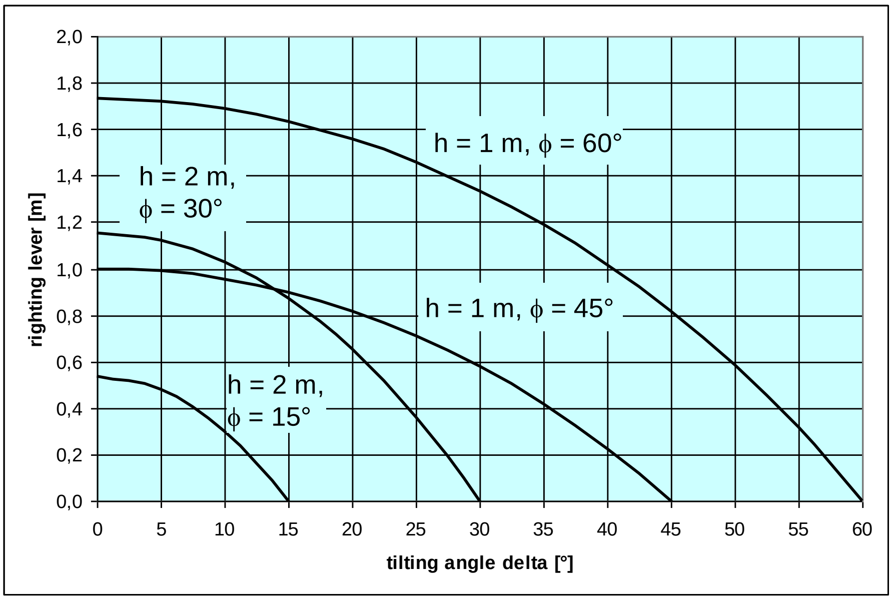


Figure 1.16: Range of suspension stability

1.3.2 Primary and secondary slings

Figure 1.17 shows the relevant parameters for assessing a suspension with primary and secondary slings.

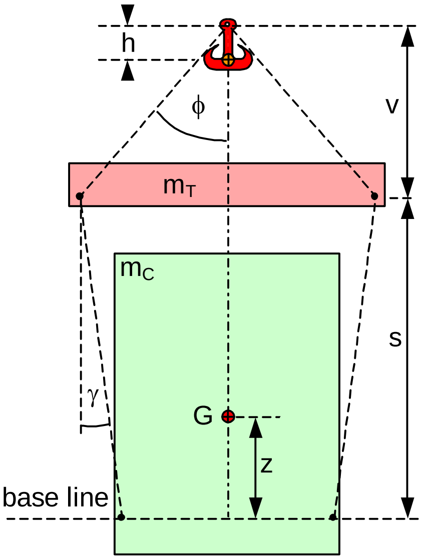


Figure 1.17: Parameters of a primary and secondary suspension

Metacentric height

$$h = s \cdot (1 - c) + v \cdot \left(1 + \frac{m_T}{m_C}\right) - z \cdot \left(1 - \frac{c \cdot s \cdot \tan\gamma}{v \cdot \tan\varphi + s \cdot \tan\gamma}\right) \text{ [m]}$$

$$c = \cos^2\gamma - \left(1 + \frac{m_T}{m_C}\right) \cdot \frac{\sin\gamma \cdot \cos\gamma}{\tan\varphi}$$

The formulas for obtaining c and h are inconvenient and prone to typing errors in manual computer calculation. It is therefore recommended to apply a simple Excel sheet as shown below. The green shaded cells are entry data, the orange shaded cells contain the results.

	A	B	C	D	E	F	G	H
1	s	v	z	phi	gamma	m_T	m_C	h
2	8,46	4,56	2,66	42,0	-7,0	18,0	64,0	0,809
3			c	phi rad	gamma rad			
4			1,15727	0,73304	-0,12217			
5								

c: C4=cos(E4)^2-(1+F2/G2)*sin(E4)*cos(E4)/tan(D4)

ϕ_{rad}: D4=D2*PI()/180

γ_{rad}: E4=E2*PI()/180

h: H2=B2*(1+F2/G2)+A2*(1-C4)-C2*(1-C4*A2*tan(E4)/(B2*tan(D4)+A2*tan(E4)))

If the secondary suspension is exactly vertical, i.e. angle $\gamma = 0$ with c = 1, the formula for the metacentric height is simplified to read:

$$h = v \cdot \left(1 + \frac{m_T}{m_C}\right) - z \ \ [m]$$

Soft slings

The use of soft slings in a combined primary and secondary suspension will create similar impairment to the metacentric height as shown for pure primary suspensions. It is therefore recommended not to use soft slings in such combined arrangements, unless an ample figure of positive metacentric height can be verified.

1.3.3 Asymmetric suspension arrangements

In case of asymmetric arrangements the suspension on both ends of the cargo unit should be analysed separately with the results of the partial masses m_1 and m_2 and the associated metacentric heights h_1 and h_2. The common metacentric height is obtained by:

$$h = \frac{m_1 \cdot h_1 + m_2 \cdot h_2}{m} \ \ [m]$$

h = common metacentric height [m]
h_1 = front metacentric height [m]
h_2 = rear metacentric height [m]
m = total mass of cargo unit [t]
m_1 = partial mass of cargo unit at front end [t]
m_2 = partial mass of cargo unit at rear end [t]

2. Mathematical analysis

2.1 Vertical secondary suspension

Figure 2.1 shows a suspension arrangement with primary and secondary slings. The mass of the spreader is ignored.

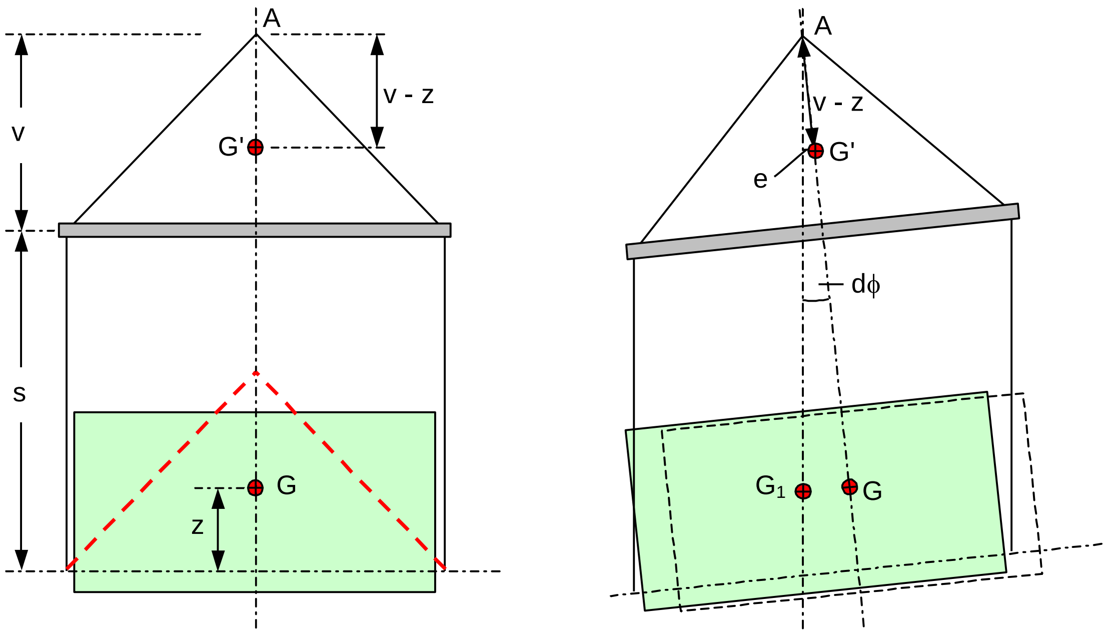


Figure 2.1: Virtual position of the c.o.g with ignored mass of spreader

In the left part of Figure 2.1 the cargo unit hangs straight. In the right part it is tilted by the small angle $d\phi$ caused by an initially unknown eccentricity e of the centre of gravity. This eccentricity shall be determined as the difference between the total shift of the centre of gravity GG_1 and the slewing of the cargo unit $s \cdot d\phi$, caused by the secondary suspension.

$$GG_1 = (v + s - z) \cdot d\varphi \quad [m]$$

$$e = (v + s - z) \cdot d\varphi - s \cdot d\varphi = (v - z) \cdot d\varphi \quad [m] \tag{1}$$

The distance (v – z) below the centre of suspension leads to a point, where the eccentricity e alone would create the new condition of equilibrium. This point is the "virtual" centre of gravity of the cargo unit within the suspension arrangement. The effective "metacentric height" of the suspension is:

$$h = v - z \quad [m] \tag{2}$$

This leads to the well-known practical rule: "Draw the primary suspension down to the base level of the secondary suspension. As long as the real centre of gravity is within the shifted triangle of the primary suspension, the cargo hangs stable:"

The above practical rule is not quite correct as it ignores the stabilising influence of the spreader. This influence is investigated below where the common centre of gravity G* of both the cargo unit and the spreader is considered.

It should be noted that the influence of the mass of the spreader appears in the results below as a relation in the form of m_T / m_C. In case of a symmetrical suspension on both ends of the cargo unit, m_T is the mass of both spreaders and m_C is the full mass of the cargo unit. In case of asymmetric arrangements, where both ends of the cargo unit will be considered separately, m_T is the mass of the considered spreader and m_C is the partial mass carried by the considered arrangement.

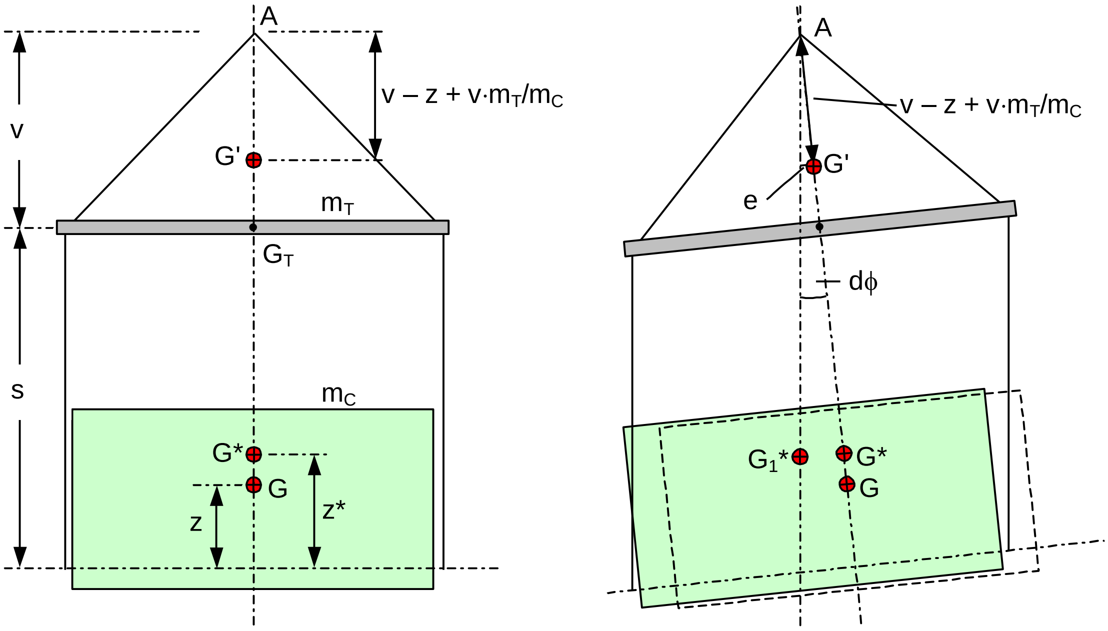


Figure 2.2: Virtual position of the c.o.g. with consideration of the mass of the spreader

The level z* of the common centre of gravity G* of the cargo unit and the spreader is determined by:

$$z^* = \frac{m_C \cdot z + m_T \cdot s}{m_C + m_T} \text{ [m]} \tag{3}$$

The common centre G* moves in the tilted condition to G_1*. The distance of this movement is:

$$G^* G_1^* = (v + s - z^*) \cdot d\varphi \text{ [m]}$$

This distance is also the unknown eccentricity e of the centre of gravity G plus the slewing distance s · dϕ from the secondary suspension. The eccentricity e may be determined by equalisation with:

$$G^* G_1^* = \frac{m_C \cdot (e + s \cdot d\varphi)}{m_C + m_T} \text{ [m]} \tag{4}$$

The solution is:

$$e = \left(v - z + v \cdot \frac{m_T}{m_C} \right) \cdot d\varphi \text{ [m]} \tag{5}$$

The virtual centre of gravity is lower by the amount of v · m_T/m_C than without considering the spreader mass. The effective "metacentric height" of the suspension is:

$$h = v \cdot \left(1 + \frac{m_T}{m_C} \right) - z \text{ [m]} \tag{6}$$

2.2 Inclined secondary suspension

In the previous considerations the secondary slings hang vertically and parallel. Consequently the tilting angle of the whole arrangement is always equal to the tilting angle of the secondary slings, namely dϕ. With an inclined secondary suspension the slewing angles dγ of the secondary suspension are different from dϕ and cause an additional tilting of the cargo unit. This will additionally influence the level of the virtual centre of gravity in either way, up or down, depending on the sign of γ.

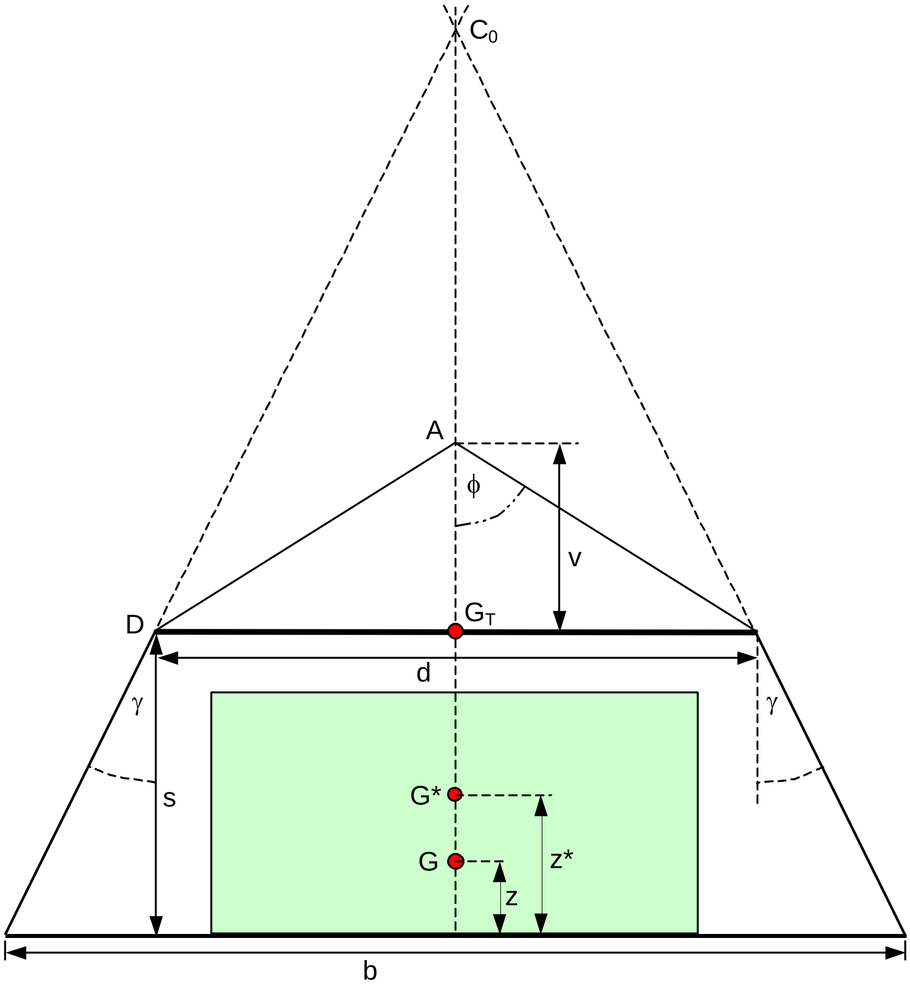


Figure 2.3: Arrangement with a positive angle γ of the secondary suspension

Figure 2.3 shows an arrangement with non-vertical secondary suspension at a positive angle γ with the centre of suspension at point A and the common centre of gravity of spreader and cargo unit at point G*. The point C_0 is the imaginary centre of the secondary suspension.

With an unknown offset e of the centre of gravity the arrangement is tilted about the point A by the small angle $d\phi$ (see Figure 2.4). The common centre of gravity G^*_1 settles necessarily below the centre of suspension A. The cargo centre of gravity G_1 is in line with G^*_1 and G_T at a distance which reflects the inverse proportionality as follows:

$$\frac{\overline{G_1 * G_1}}{\overline{G_1 * G_T}} = \frac{m_T}{m_C} \tag{7}$$

The imaginary centre of the secondary suspension C is vertically above G_1 forming the triangle BCD (shaded light blue). The secondary suspension has consequently tilted by the angle $d\gamma$. The magnitude of $d\gamma$ may be related to the magnitude $d\phi$ by the relation:

$$d\gamma = c \cdot d\varphi \ \text{[rad]} \tag{8}$$

The triangle BCD contains both the angles $d\gamma$ and $d\phi$ and is used for the determination of the factor c by means of the rule of sine's.

distance DB = $d/2 - (1 + m_T/m_C) \cdot v \cdot d\phi$
distance BC = $d / (2 \cdot \tan\gamma)$ (Note: BC = G_TC_0 with $d\phi \to 0$)
angle BCD = $\gamma + d\gamma - d\phi$
angle CDB = $90° - \gamma - d\gamma$

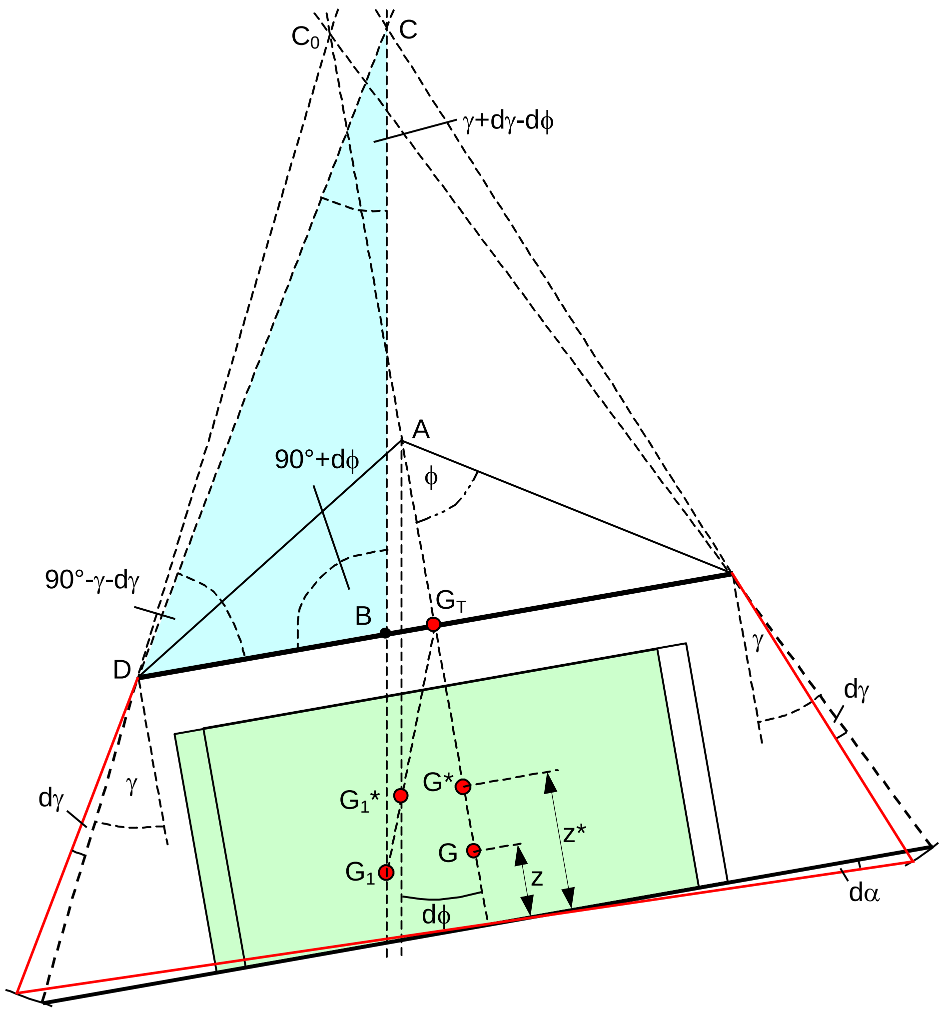


Figure 2.4: Arrangement tilted by dϕ due to an offset of G

Rule of sine's:

$$\frac{\overline{BC}}{\overline{DB}} = \frac{\sin(90° - \gamma - d\gamma)}{\sin(\gamma + d\gamma - d\varphi)} \tag{9}$$

$$\frac{d}{2 \cdot \tan\gamma \cdot (d/2 - (1 + \frac{m_T}{m_C}) \cdot v \cdot d\varphi)} = \frac{\sin(90° - \gamma - c \cdot d\varphi)}{\sin(\gamma + c \cdot d\varphi - d\varphi)} = \frac{\cos(\gamma + c \cdot d\varphi)}{\sin(\gamma - (1-c) \cdot d\varphi)}$$

$$\frac{d}{d \cdot \tan\gamma \cdot (1 - (1 + \frac{m_T}{m_C}) \cdot \frac{2 \cdot v}{d} \cdot d\varphi)} = \frac{\cos\gamma - c \cdot \sin\gamma \cdot d\varphi}{\sin\gamma - (1-c) \cdot \cos\gamma \cdot d\varphi} \tag{11}$$

$$\frac{1}{\tan\gamma - (1 + \frac{m_T}{m_C}) \cdot \frac{\tan\gamma}{\tan\varphi} \cdot d\varphi} = \frac{\cos\gamma - c \cdot \sin\gamma \cdot d\varphi}{\sin\gamma - (1-c) \cdot \cos\gamma \cdot d\varphi}$$

$$\sin\gamma - (1-c) \cdot \cos\gamma \cdot d\varphi = \sin\gamma - c \cdot \frac{\sin^2\gamma}{\cos\gamma} \cdot d\varphi - (1 + \frac{m_T}{m_C}) \cdot \frac{\sin\gamma}{\tan\varphi} \cdot d\varphi + k \cdot d^2\varphi$$

$$-\cos\gamma + c \cdot \cos\gamma = -c \cdot \frac{\sin^2\gamma}{\cos\gamma} - (1 + \frac{m_T}{m_C}) \cdot \frac{\sin\gamma}{\tan\varphi}$$

$$c \cdot \left(\cos\gamma + \frac{\sin^2\gamma}{\cos\gamma} \right) = \cos\gamma - (1 + \frac{m_T}{m_C}) \cdot \frac{\sin\gamma}{\tan\varphi}$$

$$c \cdot \frac{1}{\cos\gamma} = \cos\gamma - \left(1 + \frac{m_T}{m_C}\right) \cdot \frac{\sin\gamma}{\tan\varphi}$$

$$c = \cos^2\gamma - \left(1 + \frac{m_T}{m_C}\right) \cdot \frac{\sin\gamma \cdot \cos\gamma}{\tan\varphi} \qquad (10)$$

This solution shows that c depends from γ and also from ϕ.

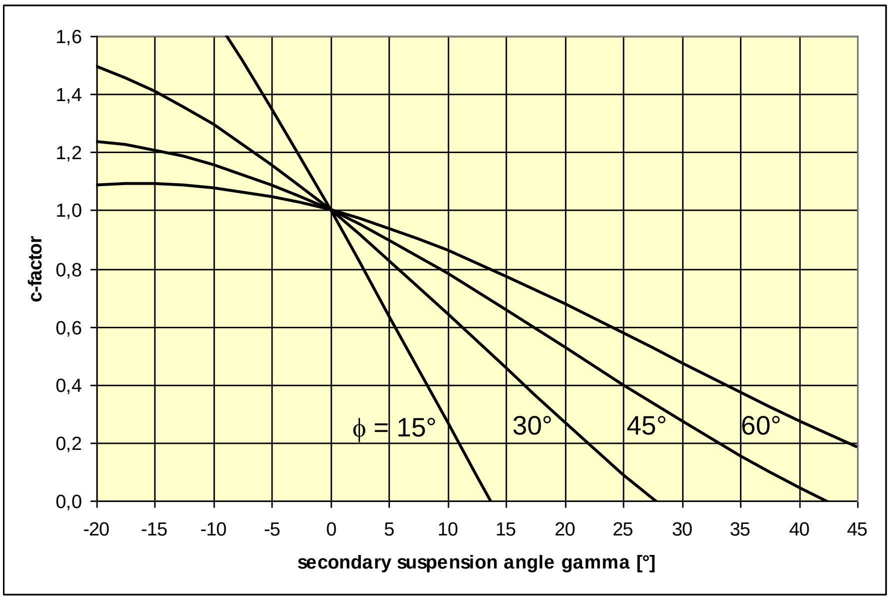


Figure 2.5: Factor c for $m_T/m_C = 0.1$ and ϕ = 15°, 30°, 45°, 60°

The slewing of the secondary suspension causes a horizontal shift of the cargo unit and also an additional tilting. This is shown by the red lines in Figure 2.4.

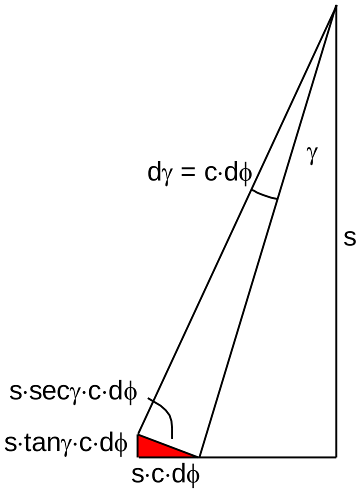


Figure 2.6: Shifting and tilting due to slewing of the secondary suspension

The tilting about the angle $d\gamma$ causes a shifting of the cargo unit by the distance ($s \cdot c \cdot d\phi$) to the **left**. This is associated with a lifting of the left side by ($s \cdot \tan\gamma \cdot c \cdot d\phi$) and a lowering of the right side by the same distance. This causes the cargo unit to tilt by the angle $d\alpha$ to the **right**. This applies for a **positive** angle γ, when the offset e is directed to the **left**.

If the centre of gravity G is positioned by the distance z above the slinging level, it will also be moved to the **right** by the distance $z \cdot d\alpha$. The angle $d\alpha$ is found by:

$$d\alpha = \frac{2 \cdot c \cdot s \cdot \tan\gamma \cdot d\varphi}{b} \text{ [rad]} \tag{11}$$

The position of the virtual centre of gravity is determined by means of the still unknown eccentricity e as follows:

$$\overline{G^*G^*_1} = (v + s - z^*) \cdot d\varphi = (v + s - \frac{z \cdot m_C + s \cdot m_T}{m_C + m_T}) \cdot d\varphi \text{ [m]} \tag{12}$$

$$\overline{G^*G^*_1} = \frac{m_C \cdot (c \cdot s \cdot d\varphi - \frac{z \cdot 2 \cdot c \cdot s \cdot \tan\gamma \cdot d\varphi}{b} + e)}{m_C + m_T} \text{ [m]}$$

$$((v+s) \cdot m_C + v \cdot m_T + s \cdot m_T - z \cdot m_C - s \cdot m_T) \cdot d\varphi = m_C \cdot (c \cdot s \cdot d\varphi - \frac{2 \cdot z \cdot c \cdot s \cdot \tan\gamma \cdot d\varphi}{b} + e)$$

$$(v + s + v \cdot \frac{m_T}{m_C} - z) \cdot d\varphi = c \cdot s \cdot d\varphi - c \cdot z \cdot \frac{b-d}{b} \cdot d\varphi + e \text{ [m]}$$

$$e = (v \cdot (1 + \frac{m_T}{m_C}) + s \cdot (1-c) - z \cdot (1 - c \cdot \frac{b-d}{b})) \cdot d\varphi \text{ [m]} \tag{13}$$

The effective "metacentric height" of the suspension arrangement is:

$$h = v \cdot (1 + \frac{m_T}{m_C}) + s \cdot (1-c) - z \cdot (1 - c \cdot \frac{b-d}{b}) \text{ [m]} \tag{14}$$

This formula may be converted by means of the relations $d = 2 \cdot v \cdot \tan\phi$ and $b = d + 2 \cdot s \cdot \tan\gamma$.

$$h = v \cdot (1 + \frac{m_T}{m_C}) + s \cdot (1-c) - z \cdot (1 - c \cdot \frac{s \cdot \tan\gamma}{v \cdot \tan\varphi + s \cdot \tan\gamma}) \quad \text{[m]} \tag{15}$$

2.3 Flexible primary suspensions

The previous analyses were taken under the assumption that the length of slings under load is always stationary, i.e. their primary elongation will not change due to small changes in the load when the hanging cargo unit is tilted. This is certainly justified with wire rope slings, which have an elongation that is governed by a modulus of elasticity of around 10^4 kN/cm^2. However, the elastic elongation of long polyester slings is much greater and the secondary effect of changes in elongation due to tilting of the cargo unit may contribute to an additional loss of suspension stability.

Figure 2.7 shows a dual crane lift with two identical primary suspensions using wire rope grommets. This is not critical in general. It might, however, become critical, if the wire rope grommets were replaced by polyester grommets.

The following analysis is directed to the primary suspension of a cargo unit with the fixing points of the slings below the centre of gravity G. The slings have a distinguished elasticity so that the originally identical length becomes unequal under the influence of unequal loads. The analysis is carried out for the longitudinal view and for the transverse view.

As the essential interest of the analysis is directed to parameters influencing the stability of the suspension, it is assumed that the arrangement is symmetrical in both the longitudinal and the transverse view. The forces in the slings are therefore identical in the initial condition.

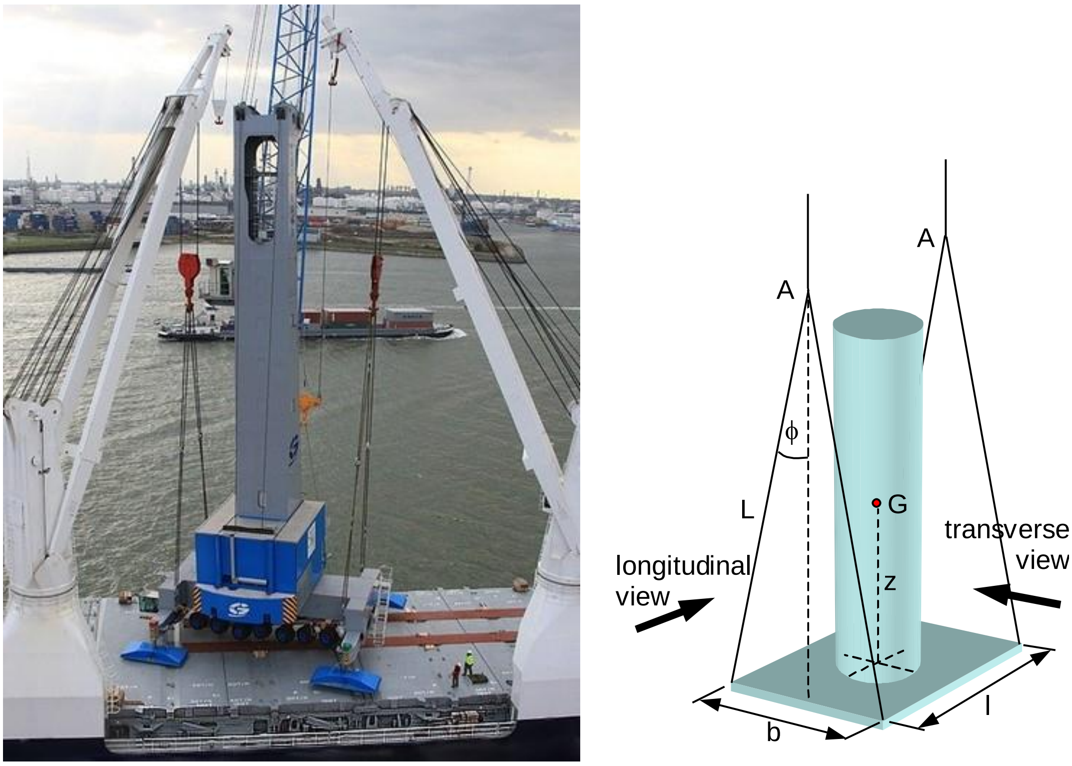


Figure 2.7: Dual crane lift with primary suspension, schematic presentation

Longitudinal view

The originally straight suspension is tilted by a small angle $d\phi$ caused by an initially unknown eccentricity e of the cargo centre of gravity. Due to the change of load in the slings the base b of the cargo unit is additionally tilted by the angle $d\alpha$.

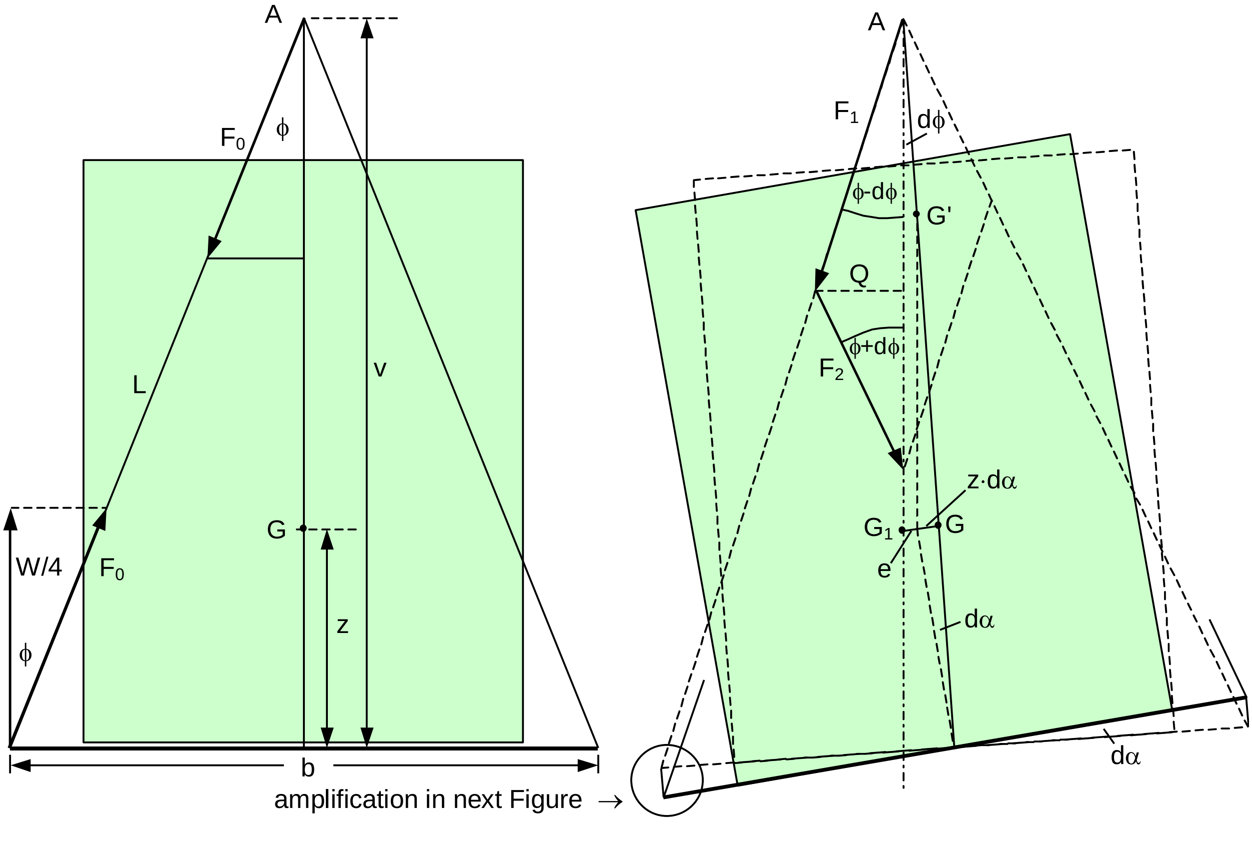


Figure 2.8: Primary suspension with elastic elongation differences in slings

This additional inclination creates an additional shift of the centre of gravity, if it is situated above (or below) the slinging base b. The effect of this additional shift is already contained in the observed tilting angle $d\phi$.

In the upright condition (Figure 2.8 left) the force in each sling is determined by the total weight $W = m \cdot g$ of the cargo unit under the assumption of 4 symmetrical slings:

$$F_0 = \frac{W}{4} \cdot \frac{1}{\cos\varphi} \text{ [kN]} \tag{16}$$

The determination of the forces F_1 and F_2 in the tilted condition shall identify the relation of the force difference dF to the tilting angle $d\phi$.

$$Q = F_1 \cdot \sin(\varphi - d\varphi) = F_2 \cdot \sin(\varphi + d\varphi) \quad \text{[kN]}$$

$$W/2 = F_1 \cdot \cos(\varphi - d\varphi) + F_2 \cdot \cos(\varphi + d\varphi) \text{ [kN]}$$

$$F_2 = \frac{W/2 - F_1 \cdot \cos(\varphi - d\varphi)}{\cos(\varphi + d\varphi)} \text{ [kN]}$$

$$F_1 \cdot \sin(\varphi - d\varphi) = \frac{W/2 - F_1 \cdot \cos(\varphi - d\varphi)}{\cos(\varphi + d\varphi)} \cdot \sin(\varphi + d\varphi) \text{ [kN]}$$

$$F_1 \cdot \left(\sin(\varphi - d\varphi) \cdot \cos(\varphi + d\varphi) + \cos(\varphi - d\varphi) \cdot \sin(\varphi + d\varphi)\right) = W \cdot \sin(\varphi + d\varphi)/2$$

$$F_1 \cdot \sin\left((\varphi - d\varphi) + (\varphi + d\varphi)\right) = F_1 \cdot \sin(2 \cdot \varphi) = W \cdot \sin(\varphi + d\varphi)/2$$

$$F_1 \cdot 2 \cdot \sin\varphi \cdot \cos\varphi = W \cdot \sin(\varphi + d\varphi)/2 = W \cdot \left(\sin\varphi \cdot \cos d\varphi + \cos\varphi \cdot \sin d\varphi\right)/2$$

$$F_1 \cdot 2 \cdot \sin\varphi \cdot \cos\varphi = W \cdot \left(\sin\varphi + \cos\varphi \cdot d\varphi\right)/2; \quad \textbf{note: } \cos d\varphi = 1; \sin d\varphi = d\varphi$$

Similarly: $$F_2 \cdot 2 \cdot \sin\varphi \cdot \cos\varphi = W \cdot \left(\sin\varphi - \cos\varphi \cdot d\varphi\right)/2$$

$$F_1 = \frac{W}{4} \cdot \frac{(\sin\varphi + \cos\varphi \cdot d\varphi)}{\sin\varphi \cdot \cos\varphi} \quad \text{und} \quad F_2 = \frac{W}{4} \cdot \frac{(\sin\varphi - \cos\varphi \cdot d\varphi)}{\sin\varphi \cdot \cos\varphi} \text{ [kN]} \tag{17}$$

$$dF = F_1 - F_0 \quad \text{and} \quad dF = F0 - F_2 \text{ [kN]}$$

$$dF = \frac{W}{4} \cdot \frac{\sin(\varphi \pm d\varphi)}{\sin\varphi \cdot \cos\varphi} - \frac{W}{4} \cdot \frac{1}{\cos\varphi} = \frac{W}{4} \cdot \frac{\sin\varphi \pm \cos\varphi \cdot d\varphi - \sin\varphi}{\sin\varphi \cdot \cos\varphi} \text{ [kN]}$$

$$dF = \pm\frac{W}{4} \cdot \frac{d\varphi}{\sin\varphi} \text{ [kN]} \tag{18}$$

The force difference dF causes a change of length dL in the slings. This change of length may be conveniently determined by the nominal spring constant D_N of the slings.

$$dL = \frac{dF \cdot L}{D_N} \text{ [m]} \tag{18}$$

The nominal spring constant D_N of a lifting sling may be determined by:

$$D_N = \frac{\Delta F}{\varepsilon} \text{ [kN]} \tag{20}$$

Test runs with polyester grommets have shown an $\varepsilon = 0.023$ for $\Delta F = WLL$. Allowing the load of WLL for a lifting operation, the applicable D_N is obtained by:

$$D_N = \frac{W}{4 \cdot \varepsilon \cdot \cos\varphi} \text{ [kN]} \tag{21}$$

Due to the change in length dL the base of the cargo unit is tilted by the angle $d\alpha$.

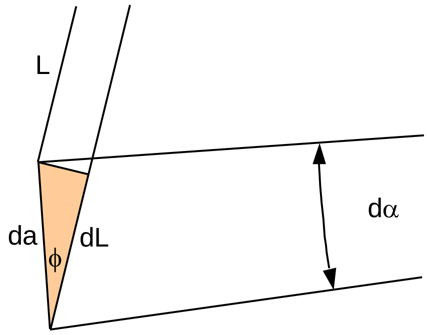


Figure 2.9: Tilting angle dα due to elongation changes of slings

$$d\alpha = \frac{2 \cdot da}{b} = \frac{2 \cdot dL}{b \cdot \cos\varphi} = \frac{2 \cdot dF \cdot L}{D_N \cdot b \cdot \cos\varphi} = \frac{2 \cdot L \cdot \varepsilon}{b \cdot \sin\varphi} \cdot d\varphi \ \text{[rad]}$$

$$d\alpha = \frac{4 \cdot L^2 \cdot \varepsilon}{b^2} \cdot d\varphi \quad \text{[rad]} \tag{22}$$

The secondary transverse movement of the centre of gravity due to tilting by the angle dα is equal to z · dα with z = elevation of G above the slinging base b. The observed total tilting angle dϕ permits to find the initially unknown offset e and finally the metacentric height, which allows the judgement of the suspension stability.

$$e + z \cdot d\alpha = (v - z) \cdot d\varphi \qquad \text{[m]} \tag{23}$$

$$e = (v - z) \cdot d\varphi - z \cdot d\alpha$$

$$e = \left(v - z \cdot \left(1 + \frac{4 \cdot L^2 \cdot \varepsilon}{b^2} \right) \right) \cdot d\varphi \ \text{[m]} \tag{24}$$

$$h = v - z \cdot \left(1 + \frac{4 \cdot L^2 \cdot \varepsilon}{b^2} \right) \ \text{[m]} \tag{25}$$

The metacentric height h is reduced with growing values of L and ε and with decreasing values of b.

Transverse view

There is no defined common centre of suspension in the transverse view. An initially unknown eccentricity e of the cargo centre of gravity causes a change of loads in the suspensions, namely an increase on the left side and a decrease on the right side in Figure 2.10. The slings react with elongation on the left side and shortening on the right side. This creates a tilting of the base of the cargo unit which amplifies the transverse movement e of the centre of gravity by the distance z · dα. The total distance (e + z · dα) is finally responsible for the change of loads. In the upright condition the forces in all slings are determined by the weight W = m · g of the cargo unit:

$$F_0 = \frac{W}{4} \cdot \frac{1}{\cos\varphi} \ \text{[kN]} \tag{16}$$

In the tilted condition the forces in the lifting tackles H are:

$$H_{left} = W \cdot \left(\frac{l/2 + e + z \cdot d\alpha}{l} \right) = \frac{W}{2} \cdot \left(1 + \frac{2 \cdot (e + z \cdot d\alpha)}{l} \right) \ \text{[kN]}$$

$$H_{right} = W \cdot \left(\frac{l/2 - e - z \cdot d\alpha}{l} \right) = \frac{W}{2} \cdot \left(1 - \frac{2 \cdot (e + z \cdot d\alpha)}{l} \right) \ \text{[kN]}$$

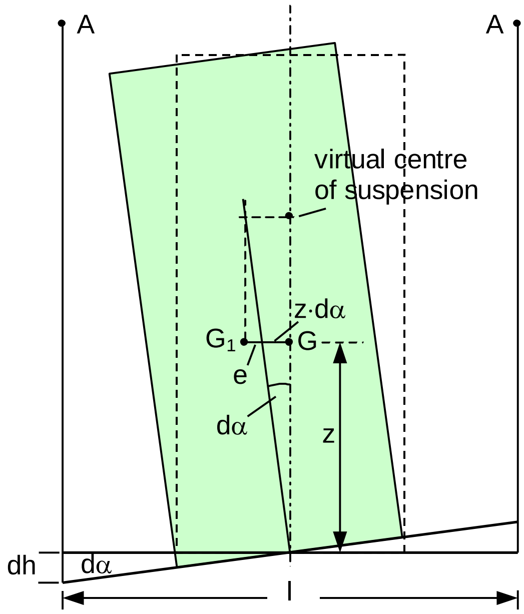


Figure 2.10: Dual crane lift in transverse view

The forces in the slings are:

$$F = \frac{W}{4 \cdot \cos\varphi} \cdot \left(1 \pm \frac{2 \cdot (e + z \cdot d\alpha)}{l}\right) \text{ [kN]} \tag{26}$$

The force differences are:

$$dF = \pm \frac{W \cdot (e + z \cdot d\alpha)}{2 \cdot l \cdot \cos\varphi} \text{ [kN]} \tag{27}$$

The elongations in the slings are:

$$dL = \pm \frac{dF \cdot L}{D_N} = \pm \frac{W \cdot (e + z \cdot d\alpha) \cdot L}{2 \cdot D_N \cdot l \cdot \cos\varphi} \text{ [m]} \tag{28}$$

The vertical components of these elongations are:

$$dh = \pm dL \cdot \cos\varphi = \pm \frac{W \cdot (e + z \cdot d\alpha) \cdot L}{2 \cdot D_N \cdot l} \text{ [m]} \tag{29}$$

The distance dh can be expressed also by:

$$dh = \pm \frac{l}{2} \cdot d\alpha \text{ [m]} \tag{30}$$

This permits to determine the initially unknown offset e:

$$\frac{W \cdot L \cdot (e + z \cdot d\alpha)}{2 \cdot D_N \cdot l} = \frac{l \cdot d\alpha}{2} \text{ [m]}$$

$$e = \left(\frac{l^2 \cdot D_N}{W \cdot L} - z\right) \cdot d\alpha \text{ [m]} \tag{31}$$

With the nominal spring constant defined above the metacentric height is:

$$h = \frac{l^2}{4 \cdot v \cdot \varepsilon} - z \text{ [m]} \tag{32}$$

The metacentric height h is reduced with growing values of v and ε and with decreasing values of l.

2.4 Asymmetric arrangements

The analyses in the previous chapters were directed to arrangements with identical configurations of slings and spreaders on both ends of the cargo unit. However, there are quite often arrangements with different configurations on both ends, having e.g.:

- different levels of lift points on the cargo unit,
- different levels of centres of suspension,
- different types and layouts of suspension.

Figure 2.11 shows an asymmetric suspension with a single primary sling fixed below the cargo centre of gravity on the left side and a combined primary/secondary suspension fixed above the centre of gravity on the right side. The arrangement is obviously stable, but the safety margin is unknown.

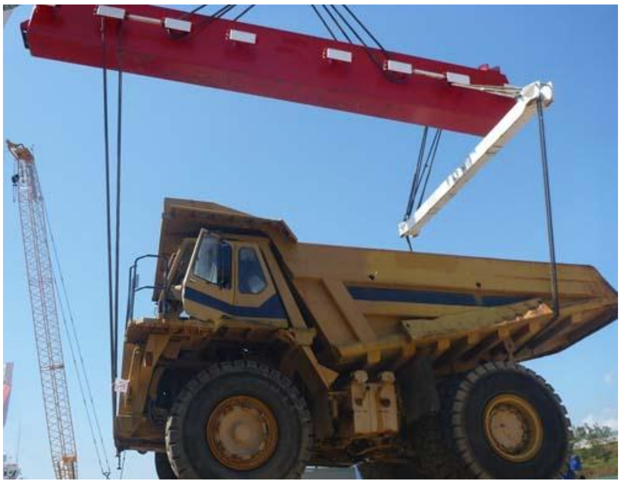


Figure 2.11: Asymmetric suspension

Figure 2.12 shows the principles of the asymmetric suspension in Figure 2.11.

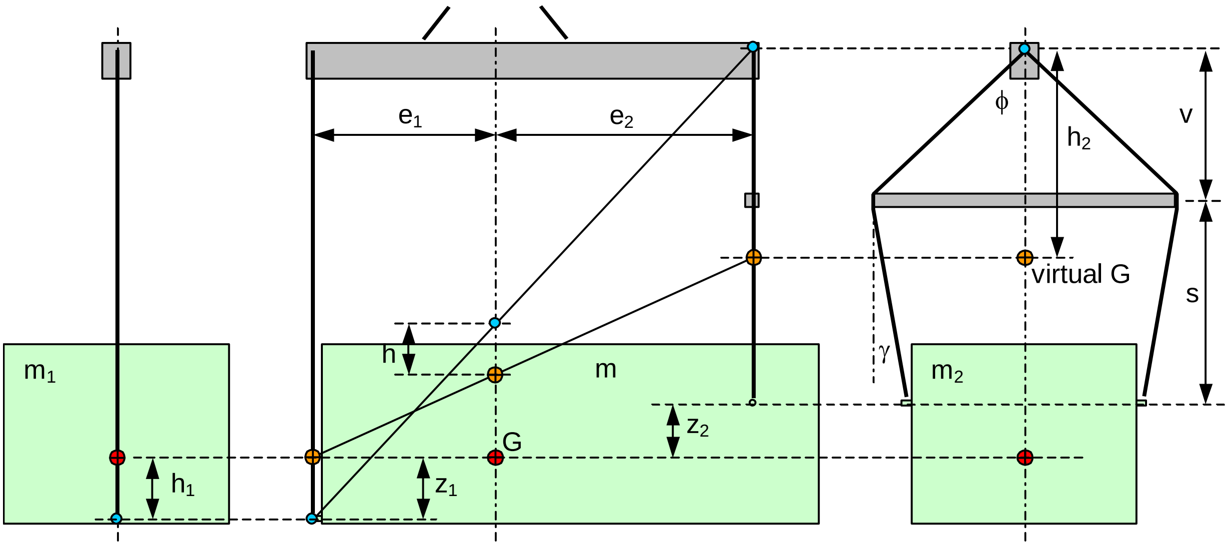


Figure 2.12: Asymmetric suspension

The left side carries the partial mass m_1 and the right side the partial mass m_2. The partitioning follows the rule of inverse proportionality:

$$m_1 = m \cdot \frac{e_2}{e_1 + e_2} \qquad \text{and} \qquad m_2 = m \cdot \frac{e_1}{e_1 + e_2} \tag{33}$$

Left side: The single primary sling will be definitely vertical in the transverse plane. If fixed to the cargo unit below its centre of gravity, it will create an instable suspension with the **negative** metacentric height h_1. If the cargo unit is tilted by the small angle $d\phi$, a negative stabilising moment will be produced.

$$M_1 = m_1 \cdot g \cdot h_1 \cdot d\varphi \ \text{[kN·m]} \tag{34}$$

Right side: The suspension with a spreader will cause the effect discussed in chapter 2.2. The centre of gravity of the partial mass m_2 is raised to a virtual position with a still positive metacentric height h_2. If the cargo unit is tilted by the small angle $d\phi$, a positive stabilising moment will be produced.

$$M_2 = m_2 \cdot g \cdot h_2 \cdot d\varphi \ \text{[kN·m]} \tag{35}$$

The algebraic sum of both moments is equal to the moment of the total mass m with the lever coming from the common metacentric height h.

$$M_1 + M_2 = m \cdot g \cdot h \cdot d\varphi \ \text{[kN·m]}$$

The sign of the common metacentric height h decides on the stability of the whole suspension.

$$h = \frac{m_1 \cdot h_1 + m_2 \cdot h_2}{m} \ \text{[m]} \tag{36}$$

Figure 2.12 contains also a **geometrical approach** for obtaining the common metacentric height h. In the side view the two individual centres of suspension (blue points) are connected by a straight line and the two active centres of gravity (orange points) are connected by a straight line. These lines intersect with the vertical line through the common centre of gravity and indicate the common centre of suspension and the common virtual centre of gravity. The vertical distance between these points is the common metacentric height h. It is positive in this example. The suspension is stable.

The proof of the correctness of the geometrical solution is shown as follows:

$$h = \frac{m_1 \cdot h_1 + m_2 \cdot h_2}{m} \ \text{[m]}$$

$$h = \frac{m \cdot \frac{e_2}{e_1 + e_2} \cdot h_1 + m \cdot \frac{e_1}{e_1 + e_2} \cdot h_2}{m} \ \text{[m]}$$

$$h = \frac{h_1 \cdot e_2 + h_2 \cdot e_1}{e_1 + e_2} \ \text{[m]}$$

The last equation shows h as the weighted mean of h_1 and h_2 in the geometrical solution.